Prolegomena to Any Future Prolegomena

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Abstract

The paper by Sacks and Doyle that is the focus of this debate resembles previous papers by Cheeseman and McDermott, leaping from the observation that a specific application of some AI technology is unable to solve a problem to the conclusion that the technology itself is of limited value. We argue that claims such as this should be made more cautiously in the future, and make constructive suggestions about future qualitative physics research based on Sacks and Doyle's results.

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In [4], Sacks and Doyle raise serious concerns about the impact that existing qualitative physics work can be expected to have on AI generally. Their argument, roughly speaking, is that expert analysis is often nonqualitative in some way and that there are severe theoretical limits on the expressiveness of qualitative equations. The conclusion that they appear to be encouraging is that qualitative analysis will in the long run be of limited value to the AI enterprise.

In this response, I would like to argue two things. First, I would like to argue against the perpetual temptation to view negative results as so damning for large segments of the AI community. This is a mistake made by other *Taking Issue* authors as well. McDermott seems tempted to dismiss declarativism as a whole and nonmonotonic reasoning in particular in [3], while Cheeseman takes an absurdly pessimistic view of any nonprobabilistic approach in [1].

This is unfortunate. Problems such as that introduced by Hanks and McDermott in [2] should be viewed as opportunities to move the field forward, not to close the lid on otherwise promising approaches. The Yale shooting problem specifically has led to a variety of interesting new ideas in nonmonotonic reasoning and the formalization of action.

Could Sacks and Doyle have managed to interpret their results in a constructive way for the qualitative physics community? In the rest of this reply, I will argue that they could have, and try to present one possible constructive interpretation. Before doing so, however, let me make a disclaimer: I am an outsider to qualitative physics, and my response will be to qualitative physics only as Sacks and Doyle describe it. There is a great deal of work on extensions to SPQR that may render their central inexpressibility result invalid for one reason or another.

The essence of Sacks and Doyle's argument is an outgrowth of the fact that qualitative methods return ambiguous results when applied to a harmonic oscillator. SPQR is able to determine that the oscillator oscillates, but not that the amplitude of the oscillations gradually drops if the oscillator is damped in some way, or that the amplitude remains constant in the undamped case.

In some sense, we should not be surprised by the fact that qualitative analyses may be ambiguous. Suppose that I am given some set of differential equations D that describes some physical situation. There is also an operator S that solves these equations, so that behavior of the physical situation is "directly" described by $S(D)$. Finally, suppose that we have an operator Q that takes a collection of quantitative equations and replaces them by their qualitative versions. The crucial insights of qualitative physics are now the following:[1]

1. Instead of $S(D)$, we may only be interested in the qualitative result $Q(S(D))$.
2. It may be much cheaper to compute $Q(S(D))$ than to compute $S(D)$ itself.

[1] Dan Weld has also pointed out to me that it may be easier to come by the qualitative equations than their quantitative analogs.

Diagrammatically, the situation is like this:

$$
\begin{array}{ccc}
D & & \\
\big\downarrow{\scriptstyle S} & & \\
S(D) & \xrightarrow{Q} & Q(S(D))
\end{array}
\tag{1}
$$

Now, it might be the case that the qualitative solution could be obtained by solving the qualitative version $Q(D)$ of the original equations, assuming that $Q(S(D)) = Q(S)[Q(D)]$, or that the following diagram commutes:[2]

$$
\begin{array}{ccc}
D & \xrightarrow{Q} & Q(D) \\
\big\downarrow{\scriptstyle S} & & \big\downarrow{\scriptstyle Q(S)} \\
S(D) & \xrightarrow{Q} & Q(S(D))
\end{array}
\tag{2}
$$

The diagram (2) is well known to the qualitative community; it is also well known that this diagram in fact does *not* commute. It is true that every qualitative solution to the quantitative equations will be a solution of the qualitative equations, so that

$$Q[S(D)] \subseteq Q(S)[Q(D)] \tag{3}$$

Equality here would mean that (2) commutes; the left hand side of (3) is the qualitative version of the quantitative solution and corresponds to the path appearing in the original diagram (1). The right hand side of (3) corresponds to the other path through the diagram (2) where we apply $Q(S)$, the qualitative version of the "solve it" operator, to the qualitative differential equations given by $Q(D)$.

Unfortunately, there may be many solutions to the qualitative equations that are without quantitative analogs. The novelty in Sacks and Doyle's paper is that they present a result indicating that the expressive power of SPQR is so limited that the qualitative equations can nearly *always* be expected to introduce ambiguities of this form; the inclusion in (3) will almost always be proper. One might, for example, have hoped to recover the commutativity of a diagram such as (2) by replacing the original set of differential equations D with an equivalent set D' such that $Q(D')$ was not ambiguous; Sacks and Doyle have shown us that within the SPQR setting, this typically cannot be done.

If we want to find a variant of (2) that respects the intuitions underlying qualitative physics and is commutative, we need either to work with a different set of equations (which Sacks and Doyle have shown won't help), work with a different S operator (which is clearly not allowed), or work with a different Q operator, presumably one whose target language is more descriptive than that of SPQR.

Do such intermediate "semiqualitative" languages exist? Presumably such a language would be more descriptive than SPQR but computationally more tractable than the quantitative differential equations with which our analysis began. In the harmonic oscillator, for

[2] I am abusing notation here by writing Q for any "make qualitative" operator, whether the operand is a set of differential equation, a solution thereto or even the S operator itself. The point should be clear enough, however.

example, we have seen that a direct qualitative translation of the usual physical description fails to imply the that the amplitude of a damped oscillator falls over time. But this is at odds with my intuitive (and "semiqualitative") analysis of this system, which uses the fact that the energy is either falling or remaining constant to conclude that the amplitude of the oscillation cannot increase.

To make this somewhat more precise, note that the potential energy of a harmonic oscillator is given by

$$E_p = \frac{1}{2}kx^2$$

where k is the spring constant; it follows from this that the total energy of the system satisfies

$$E \geq \frac{1}{2}kx^2 \tag{4}$$

When I combine this with the fact that $\dot{E} \leq 0$ (energy always decreases), I can conclude that the amplitude of the oscillation is nonincreasing as well. But although (4) is a consequence of the usual description of a harmonic oscillator, (4) is not a legitimate sentence of SPQR as Sacks and Doyle describe it, since one is not allowed to compare the relative magnitudes of two analytic quantities.

In working my analysis around (4), I have done two things: First, I have used domain knowledge about physical systems to introduce an additional axiom into the original quantitative description of the system being analyzed. This axiom is superfluous from a quantitative point of view, but not from a qualitative one. Second, I have interpreted as "semiqualitative" some sentences in the resulting description that do not fit into the limited SPQR framework.

Is it easier to do these things and to then solve the resulting semiqualitative system, or to solve the original differential equations using analytic methods? Sacks and Doyle seem to be arguing in favor of a uniform commitment to the second of these approaches, but that is surely premature – there are doubtless situations where analytic methods will be the ones of choice, and equally doubtless there are situations where qualitative or semiqualitative methods are to be preferred. As an example, the conclusion that all damped oscillators have decreasing amplitudes can be made using semiqualitative methods without the need for an appeal to the precise form of the damping factor.

In fact, there is a spectrum of possible analyses. At one end of the spectrum is the purely qualitative version of the problem as described in SPQR; at the other lie the quantitative equations describing the problem in complete detail. Qualitative physicists have argued convincingly that the analysis should not always proceed using purely quantitative techniques; Sacks and Doyle are arguing – equally convincingly – that pure SPQR has too little expressive power to be of much use in practice.

So what are we to do? We can move from the quantitative approach to techniques such as parametric differential equations and bifurcation analysis (as Sacks and Doyle recommend), or we can move from SPQR toward the extended qualitative languages that we have dubbed "semiqualitative." I can see no *a priori* reason to prefer either of these approaches over the other; there are surely advantages to each.[3]

Sacks and Doyle have done the qualitative physics community a service by analyzing the theoretical limitations of SPQR so carefully. Their analysis will help to guarantee that future extensions to SPQR avoid at least some of the shortcomings inherent in the use of the restricted language. Because of this, their result and others like it should be sources of enthusiasm, both within the qualitative physics community and with regard to the impact qualitative physics can be expected to have on AI generally. Identifying the assumptions that have hitherto restricted the applicability of qualitative analysis should lead to progress, not pessimism.

Acknowledgement

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References

[1] P. Cheeseman. An inquiry into computer understanding. *Computational Intelligence*, 4, 1988.

[2] S. Hanks and D. McDermott. Nonmonotonic logics and temporal projection. *Artificial Intelligence*, 33:379–412, 1987.

[3] D. McDermott. A critique of pure reason. *Computational Intelligence*, 3:151–160, 1987.

[4] E. P. Sacks and J. Doyle. Prolegomena to any future qualitative physics. *Computational Intelligence*, 7, 1991. This issue.

[3] As an indication of this, note that qualitative or semiqualitative analyses focus on transient behavior, while quantitative or bifurcation analyses tend to focus on asymptotic behavior. I am indebted to Leo Joskowicz for this observation.