Assignment 1

COMP 2411, Session 1, 2004

Starred questions are harder.

Q1 (2 marks): For each of the following formulas, (1) restore parentheses and (2) remove as many parentheses as possible.

- $\neg\neg p \leftrightarrow (p \leftrightarrow q \vee r)$
- $\neg(\neg p \leftrightarrow p) \leftrightarrow q \vee r$
- $\neg(p \rightarrow q) \vee r \vee s \rightarrow q$
- $p \leftrightarrow (\neg p \vee q) \rightarrow (p \wedge (q \vee r))$
- $((\neg p \vee q \vee r \wedge s) \leftrightarrow (p \wedge \neg p))$

Q2 (2 marks): Verify that not all boolean operators are definable from $\rightarrow$ and $\vee$ only.

Same question with $\neg$ and $\leftrightarrow$. (Hint: Let φ be a formula built from two atomic formulas p and q using $\neg$ and $\leftrightarrow$ only. Using properties of $\leftrightarrow$, see how φ can be simplified, and shown to be logically equivalent to one of a few simple formulas where p and q occur at most once.)

Q3 (2 marks): Using semantic tableaux, show that $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ is valid.

Using semantic tableaux, find all assignments of truth values to p and r that make $\neg p \leftrightarrow p \vee r$ true.

Q4 ($\star$) (2 marks): Given two propositional formulas φ, ψ, write $\varphi < \psi$ iff $\varphi \rightarrow \psi$ is valid but $\psi \rightarrow \varphi$ is not. For example $p \wedge q < p$ and $p \leftrightarrow q < p \rightarrow q$. Show that for all formulas φ, ψ with $\varphi < \psi$, there exists a formula χ with $\varphi < \chi < \psi$. For example, if $\varphi = p \wedge q \wedge r$ and $\psi = r$ then you can take $\chi = p \wedge r$. (Hint: try to find a formula ξ such that you can 'squeeze' $\varphi \vee \xi$ between φ and ψ.)

Q5 ($\star$) (2 marks): Say that a set X of formulas is independent iff for all $\varphi \in X$, $X \setminus \{\varphi\} \not\models \varphi$. For example, $\{p, q, r\}$ is independent but $\{p, p \wedge q, r\}$ is not, because $\{p \wedge q, r\} \models p$. Show that for all sets X of formulas, there is a set Y of formulas such that Y is independent and X and Y have the same models. For example, if $X = \{p, p \wedge q, r\}$ you can take $Y = \{p \wedge q, r\}$. (Hint: if X is enumerated as $\{\psi_1, \psi_2, \psi_3 \ldots\}$ you might first want to 'throw away' all formulas in the enumeration that are logically implied by the previous ones.)