Tutorial Solution 1

COMP 2411, Session 1, 2004

Q1: If voice one was telling the truth, then it would be lying, contradiction. Hence voice one is a lying Knave, and at least one of the others is a Knight. If voice two were lying, everybody would be a Knave, which contradicts the previous conclusion that voice one does not say the truth. Hence voice two is a Knight. Since Knights always tell the truth, voice three is a Knave.

Q2:

- $(p \rightarrow \neg q) \wedge r$
- $p \vee q \vee r$
- $(p \wedge \neg q \vee r) \vee s$
- $(q \vee \neg r) \vee p \vee s$
- $(p \leftrightarrow \neg q) \leftrightarrow \neg(r \vee s)$
- $\neg\neg\neg(q \vee r) \leftrightarrow q \vee r$
- $\neg\neg\neg(q \vee r) \leftrightarrow q \leftrightarrow r$
- $((p \rightarrow q) \rightarrow r \rightarrow s) \wedge \neg p \vee r$

Q3:

- $(r \vee (\neg p \wedge q))$
- $(q \rightarrow (\neg\neg\neg p \vee r))$
- $((\neg(p \rightarrow q) \vee (r \vee s)) \rightarrow q)$
- $(p \leftrightarrow ((\neg p \vee q) \rightarrow (p \wedge (q \vee r))))$
- $((\neg p \vee (q \vee (r \wedge s))) \leftrightarrow (p \wedge \neg p))$

Q4:

- $p \vee r$ is true
- $p \wedge r$ is false
- $\neg p \wedge \neg r$ is false
- $p \leftrightarrow \neg q \vee r$ is false
- $q \vee \neg r \rightarrow p$ is true

- $q \vee p \to q \to \neg r$ is true
- $(q \leftrightarrow \neg p) \leftrightarrow p \leftrightarrow q$ is false
- $(q \to p) \to (p \to \neg r) \to \neg r \to q$ is true

Q5: If $p \to q$ is true then

- $p \vee r \to q \vee r$ is true
- $p \wedge r \to q \wedge r$ is true
- $\neg p \wedge q \leftrightarrow p \vee q$ can be either true of false

Q6: If $p \leftrightarrow q$ is false then

- $p \wedge q$ is false
- $p \vee q$ is true
- $p \to q$ can be either true of false
- $p \wedge r \leftrightarrow q \wedge r$ can be either true of false

Q7: If $p \leftrightarrow q$ is true then

- $p \wedge q$ can be either true of false
- $p \vee q$ can be either true of false
- $p \to q$ is true
- $p \wedge r \leftrightarrow q \wedge r$ is true

Q8:

- If $\neg p \vee q \to p \to \neg r$ is false then p, q and r are all true
- If $p \wedge q \leftrightarrow p \vee q$ is false and p is false then q is true
- $(p \to \neg q) \to r \to q$ is false then q is false and r is true, and p can be either true or false

Q9:

- $\neg(p \to (q \leftrightarrow \neg r)) \equiv p \wedge (q \leftrightarrow r)$
- $\neg(\neg p \vee (q \to r)) \equiv p \wedge q \wedge \neg r$
- $\neg(p \wedge (q \vee \neg r)) \equiv \neg p \vee \neg q \wedge r$

Q10: Let propositional formulas φ and ψ be given. By the definition of $\equiv$, $\varphi \equiv \psi$ holds iff φ and ψ get the same truth value in all interpretations. By the definition of $\leftrightarrow$, $\varphi \leftrightarrow \psi$ is true iff φ and ψ are either both true or both false. The claim follows immediately.

Q11: Let φ be a formula built from propositional atoms $p, p_1, \ldots, p_n$ using $\vee$ and $\wedge$ only. If all of $p, p_1, \ldots, p_n$ get the value true, then φ also takes the value true since the disjunction of two true formulas is true, and the conjunction of two true formulas is true. Hence φ is not logically equivalent to $\neg p$. Hence negation is not definable in terms of disjunction and conjunction.

Q12: It suffices to observe that $\neg p \equiv p \vee p \rightarrow \neg p$ and $p \vee q \equiv \neg(p \vee q \rightarrow (r \wedge \neg r))$.