

Tutorial Solution 5

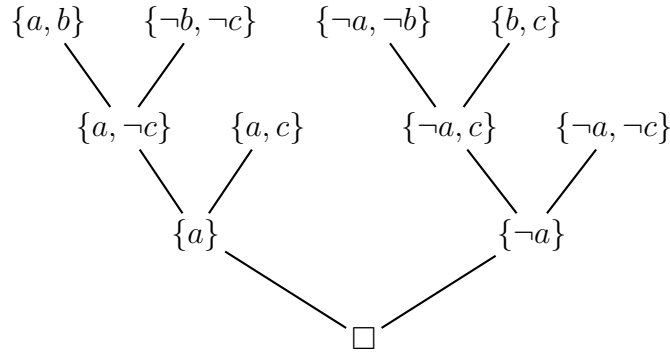
COMP 2411, Session 1, 2004

Q1: No, $\square$ is satisfiable, whereas the two resolvents of $\{p, q\}$ and $\{\neg p, \neg q\}$, namely $\{q, \neg q\}$ and $\{p, \neg p\}$, are valid.

Q2: The formula is equivalent to the set of clauses

$$\{\{a, b\}, \{b, c\}, \{a, c\}, \{\neg a, \neg b\}, \{\neg b, \neg c\}, \{\neg a, \neg c\}\}$$

It is unsatisfiable (hence its negation is valid) since the empty clause can be derived:



Q3: The formula is equivalent to the set of clauses

$$\{\{a, b\}, \{b, c\}, \{\neg a, \neg b\}, \{\neg b, \neg c\}, \{\neg a, \neg c\}\}$$

It is satisfiable (hence its negation is not valid) because we can generate the following clauses:

$$\{a, \neg c\}, \{\neg c\}, \{b\}, \{\neg a\}, \{b, \neg c\}, \{c, \neg a\}, \{b, \neg a\}$$

at which point no new clauses except valid ones (containing both an atom and its negation) can be generated.

Q4: x , c , d and e are functions symbols of arity 0, *i.e.*, constants; f is a function symbol of arity 2; g a function symbol of arity 4; P a predicate symbol of arity 1; Q a predicate symbol of arity 2.

Q5: A and B are constants; G is a function symbol of arity 1; H and P are function symbols of arity 3. Either F is a function symbol of arity 2, in which case $F(G(A), H(A, B, P(x, A, y)))$ is a term, or F is a predicate symbol of arity 2, in which case $F(G(A), H(A, B, P(x, A, y)))$ is a formula—more precisely, an atomic formula.

Q6: A and B are constants; F is a function symbol of arity 2; G is a function symbol of arity 3. Note that $F(A, G(x, B, F(y, A)))$ is necessarily a term.

Q7: One constant, say "void", and two unary function symbols, say "s0" and "s1". The sequence (0010) would be represented by the term "s0(s0(s1(s0(void))))".

Q8: A finite graph over $\mathbb{N}$ is a finite subset of $\mathbb{N}^2$. The empty graph will be represented by "void". Given $n \in \mathbb{N}$ a graph $G = \{(a_1, b_1), \dots, (a_n, b_n)\}$ consisting of n vertices will be represented by the term

$$\text{graph}(\text{arc}(\overline{a_1}, \overline{b_1}), \text{graph}(\text{arc}(\overline{a_2}, \overline{b_2}), \dots, \text{graph}(\text{arc}(\overline{a_n}, \overline{b_n}), \text{void}) \dots)$$

There are $n!$ such representations.

The term "arc(void, void)" does not naturally represent any graph.

Q9: You need predicate symbols, but you do not need function symbols.

Q10: For instance:

- $\forall x (\text{natural_number}(x) \rightarrow \exists y (s(x) = y))$
- $\neg \exists x \text{ better}(x, \text{taking_a_nap})$
- $\forall x (\text{integer}(x) \rightarrow \neg \text{negative}(x))$
- $\forall x \forall y (\text{name}(x, y) \wedge \text{innocent}(y) \rightarrow \text{changed}(y))$
- $\forall x (\text{area_of_cs}(x) \rightarrow \text{important_for}(\text{logic}, x))$
- $\forall x (\text{renter}(x) \wedge \text{in_accident}(x) \rightarrow \text{pay_deductible}(x))$
- $\text{better}(\text{bronze_medal}, \text{nothing})$
- $\neg \exists x \text{ better}(x, \text{gold_medal})$
- $\text{better}(\text{bronze_medal}, \text{gold_medal})$

Q11: All occurrences of x are bound. The first three occurrences of y are bound, the last one is free. The second and third occurrences of z are bound, the first and last one are free. A universal closure of φ is $\forall y \forall z \exists x (\forall y (P(x, y, z) \vee \exists z Q(x, y, z)) \wedge \neg R(x, y, z))$. An existential closure of φ is $\exists y \exists z \exists x (\forall y (P(x, y, z) \vee \exists z Q(x, y, z)) \wedge \neg R(x, y, z))$.

Q12: All occurrences of x , y , z and u are bound. Hence φ is closed, and identical to its universal and existential closures.

Q13: When the alphabet contains no constant.

Q14: An unmarried man is unhappy.

Some man hates every man.

Some man hates all men that hate him, and only them.