Tutorial Solutions 7

COMP 2411, Session 1, 2004

Q1: $\neg\exists x(p(x) \vee (\forall y\, q(x,y) \rightarrow \exists z\, r(x,z))) \wedge \forall y \exists z\, s(y,z) \equiv$
$\forall x(\neg p(x) \wedge \neg(\forall y\, q(x,y) \rightarrow \exists z\, r(x,z))) \wedge \forall y \exists z\, s(y,z) \equiv$
$\forall x(\neg p(x) \wedge (\forall y\, q(x,y) \wedge \forall z \neg r(x,z))) \wedge \forall v \exists w\, s(v,w) \equiv$
$\forall x \forall y \forall z \forall v \exists w(\neg p(x) \wedge q(x,y) \wedge \neg r(x,z) \wedge s(v,w))$.

Q2: For a model of $\neg\varphi_1$ and $\neg\varphi_2$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0,1\}$ and $p^{\mathfrak{M}} = \{(0,1),(1,0)\}$.

For a model of $\neg\varphi_3$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0,1\}$, $q^{\mathfrak{M}} = \{(0)\}$ and $r^{\mathfrak{M}} = \{(1)\}$.

For a model of $\neg\varphi_4$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0,1\}$, $q^{\mathfrak{M}} = \{(0)\}$ and $r^{\mathfrak{M}} = \emptyset$.

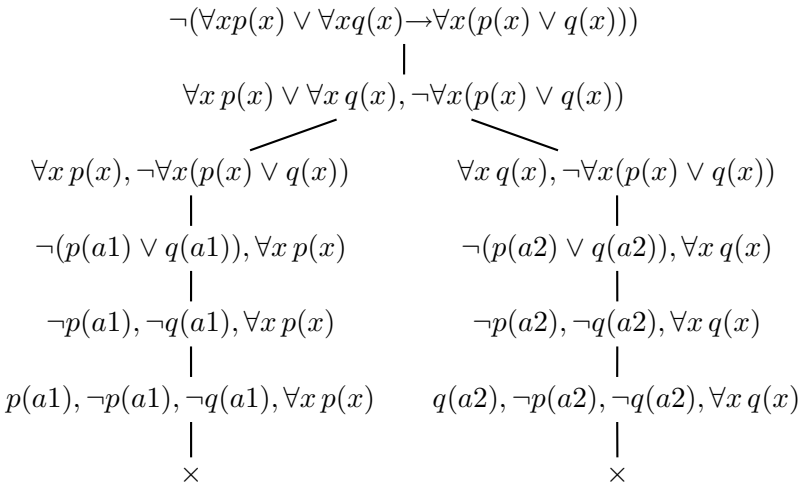
For a model of $\neg\varphi_5$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0\}$ and $p^{\mathfrak{M}} = \emptyset$.

For a model of $\neg\varphi_6$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0,1,2,3\}$ and $p^{\mathfrak{M}} = \{(0,1),(1,2),(2,3),(3,0),(0,0),(2,2)\}$.

For a model of $\neg\varphi_7$ choose $\mathfrak{M}$ with $|\mathfrak{M}| = \{0,1,2\}$ and $p^{\mathfrak{M}} = \{(0,1),(0,2),(1,2),(0,0),(1,1),(2,2)\}$.

Q3: For instance, $\exists x \neg p(x,x) \wedge \exists x \exists y(p(x,x) \wedge p(y,y) \wedge \neg p(x,y))$

Q4:



Q5:

$$\neg(\neg\exists y p(y) \rightarrow \forall y(\exists x p(x) \rightarrow p(y)))$$
$$\neg\exists y p(y), \neg\forall y(\exists x p(x) \rightarrow p(y))$$
$$\neg(\exists x p(x) \rightarrow p(a1)), \neg\exists y p(y)$$
$$\exists x p(x), \neg p(a1), \neg\exists y p(y)$$
$$p(a2), \neg p(a1), \neg\exists y p(y)$$
$$\neg p(a2), \neg p(a1), p(a2), \neg\exists y p(y)$$
$$\times$$

Q6:

$$\neg((\forall x p(x) \rightarrow \forall x q(x)) \rightarrow \forall x(p(x) \rightarrow q(x)))$$
$$\forall x p(x) \rightarrow \forall x q(x), \neg\forall x(p(x) \rightarrow q(x))$$

$\neg\forall x p(x), \neg\forall x(p(x) \rightarrow q(x))$	$\forall x q(x), \neg\forall x(p(x) \rightarrow q(x))$
$\neg p(a1), \neg\forall x(p(x) \rightarrow q(x))$	$\neg(p(a3) \rightarrow q(a3)), \forall x q(x)$
$\neg(p(a2) \rightarrow q(a2)), \neg p(a1)$	$p(a3), \neg q(a3), \forall x q(x)$
$p(a2), \neg q(a2), \neg p(a1)$	$q(a3), p(a3), \neg q(a3), \forall x q(x)$
$\odot$	$\times$

Q7:

```
solve :-
        permutation([MarcoTopping1,PepperoniTopping1,SuperTopping1,NinjaTopping1],
                    [ham,praws,salami,tuna]),
        permutation([HawaiianTopping2,PepperoniTopping2,SuperTopping2,NinjaTopping2],
                    [avocado,corn,olive,pineapple]),
        permutation([HawaiianPrice,MarcoPrice,SuperPrice,NinjaPrice],
                    [5.00,6.50,8.50,10.00]),
        discover([[_,HawaiianTopping2,HawaiianPrice],
                  [MarcoTopping1,_,MarcoPrice],
                  [PepperoniTopping1,PepperoniTopping2,_],
                  [SuperTopping1,SuperTopping2,SuperPrice],
                  [NinjaTopping1,NinjaTopping2,NinjaPrice]]),
        writef('Hawaiian pizza has mussels and %w, and costs $%d0.\n',
               [HawaiianTopping2,HawaiianPrice]),
        writef('Marco-Polo has %w and tomato, and costs $%d0.\n',
               [MarcoTopping1,MarcoPrice]),
        writef('Pepperoni has %w and %w, and costs $7.00.\n',
               [PepperoniTopping1,PepperoniTopping2]),
        writef('Super-Supreme has %w and %w, and costs $%d0.\n',
               [SuperTopping1,SuperTopping2,SuperPrice]),
        writef('Ninja-Pizza has %w and %w, and costs $%d0.\n',
               [NinjaTopping1,NinjaTopping2,NinjaPrice]).

discover(Solution) :-
        Solution = [[_,_,HawaiianPrice],
                    [MarcoTopping1,_,_],
                    _,
                    [_,SuperTopping2,_],
                    _],
        HawaiianPrice > 6.50,
        MarcoTopping1 \= ham,
        member([ham,_,8.50],Solution),
        SuperTopping2 \= pineapple,
        member([tuna,corn,Price1],Solution), Price1 =\= 6.50,
        member([Topping,olive,5.00],Solution), Topping \= salami,
        member([_,pineapple,Price2],Solution), Price2 \= 10.00.
```