

Tutorial 2

COMP 2411, Session 1, 2004

Q1: Determine whether the following formulas are satisfiable.

- $(p \vee q) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$
- $(p \rightarrow q) \vee r \leftrightarrow \neg q \wedge (p \vee r)$

A formula is in conjunctive normal form (CNF) if it is a conjunction of disjunctions of literals.

A formula is in disjunctive normal form (DNF) if it is a disjunction of conjunctions of literals.

Q2: Find formulas in conjunctive and in disjunctive normal forms that are logically equivalent to the following formulas.

- $(p \vee q) \wedge (\neg q \vee r)$
- $\neg p \vee (q \rightarrow \neg r)$
- $p \wedge \neg q \vee p \vee r$
- $p \vee q \leftrightarrow \neg r$

Q3: Given a propositional atom and a nonnull $n \in \mathbb{N}$, give an example of a set X of formulas such that:

- X contains n formulas;
- $X \models p$;
- for all $\varphi \in X$, $X \setminus \{\varphi\} \not\models p$.

Q4: Prove that a formula φ that contains $\leftrightarrow$ as its only boolean operator is valid if and only if each propositional atom that occurs in φ has an even number of occurrences in φ .

Q5: Using the tableau method, show that $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ is valid.

Q6: Using the tableau method, show that

$$\{p \rightarrow q \rightarrow r, p \wedge q\} \models r.$$

Q7: Using the tableau method, find all assignments of truth values to p , q , r and s that make the formula $(p \vee s) \wedge (\neg q \leftrightarrow r) \wedge (\neg r \vee \neg s)$ true.

Q8: Show that in the worst case, the tableau method is not more efficient than the truth table method for proving that a formula is unsatisfiable.