OBVIOUSNESS: A WINDOW ON THE MIND?

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Cognitive Judgments

A king on an empty board can reach every square.

* A knight on an empty board can reach every square.

Consider a graph with colored nodes such that every arc connects two nodes of the same color. Any two nodes connected by a path of arcs are the same color.

* It is impossible to cover a checkerboard with dominos so that only two opposite corners remain uncovered.

In a finite interval of time a bouncing ball can only bounce a finite number of times.

More Examples

Consider a row of marbles where every marble is either red or green. If the first marble is red and the last marble is green then there must be a red marble next to a green marble.

Consider a chess board. The white pawns start on the second rank and never move backward. Therefore no white pawn can ever appear on the first rank.

Consider two containers for holding marbles. Initially each container is empty. Marbles are then placed in the containers in pairs; one marble from each pair is placed in each container. At any later time there will be the same number of marbles in each container.

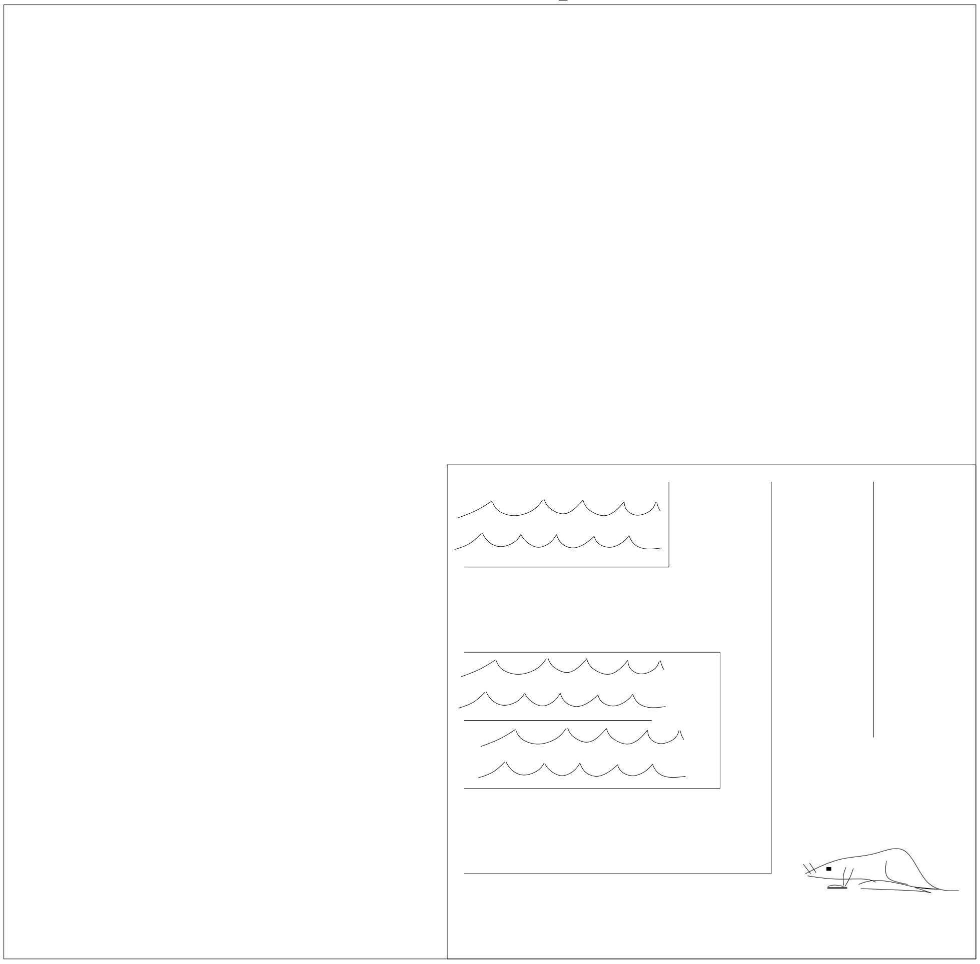
Consider Rubic's cube. Suppose the cube starts in a solved position and is scrambled by some number of rotations of faces of the cube. There exists a set of steps that unscrambles the cube.

Outline

- Against Imagery
- Kleene Star Phenomenon
- Disjointness Phenomenon
- Relational Phenomenon
- Constructive Content Phenomenon

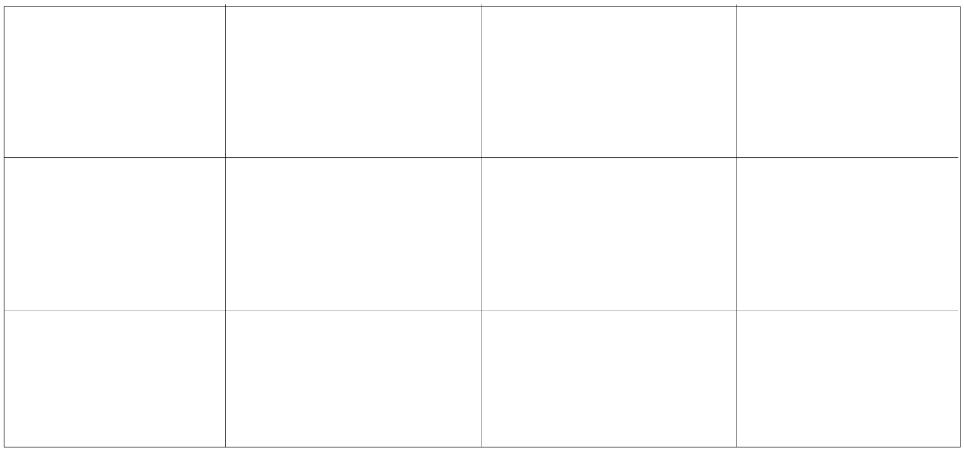
Against Imagery

Rat Example



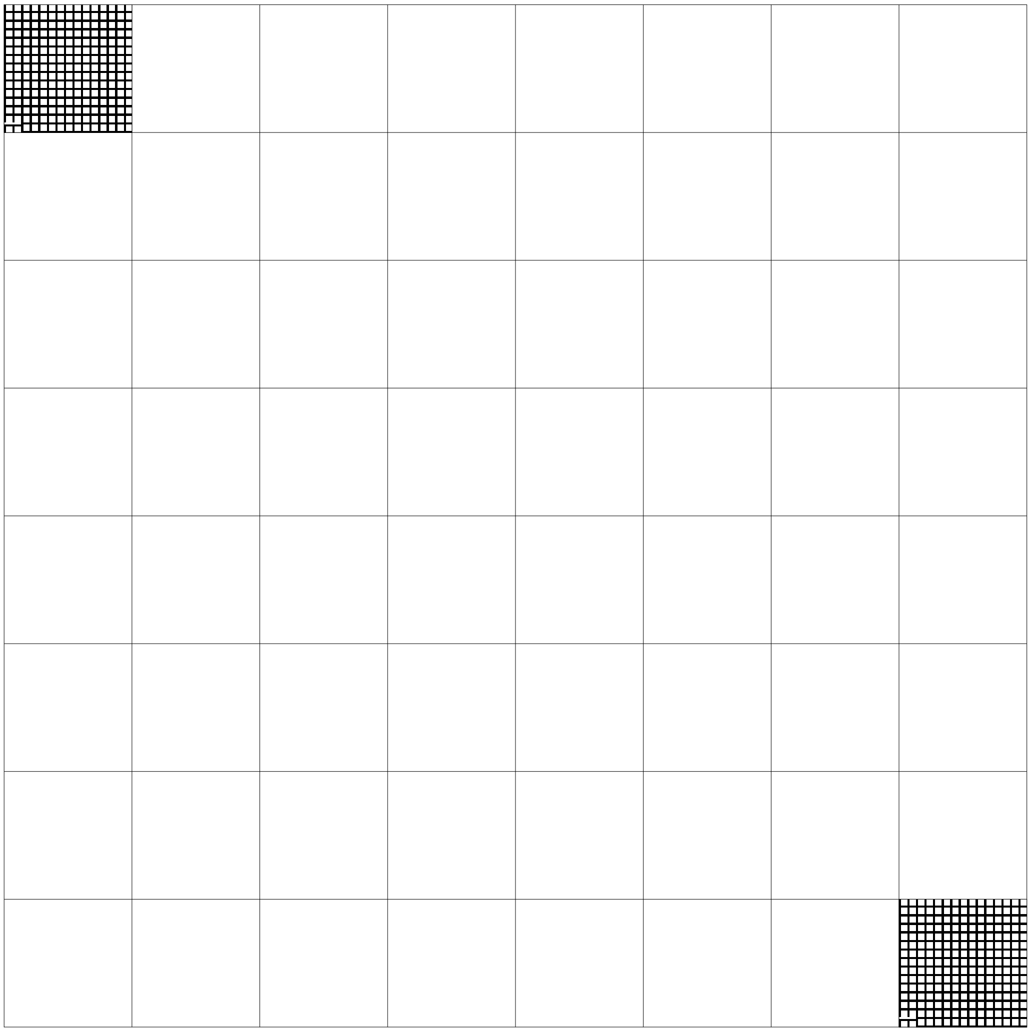
If water is filling part of a level maze with water tight walls, a dry rat can not reach the water.

Swiss Chocolate



This Swiss chocolate bar has twelve subsquares. How many breaks does it take to reduce bereak this into the twelve smaller pieces. Note that the initial breaks can be large.

Mutilated Checkerboard



Is it possible to cover a checkerboard with dominos so that opposite corners remain uncovered.

Form Matters

* Anyone who does not love fribbles is not a Zarcon. Fribble lovers are happy. So Zarcons are happy.

Zarcons love fribbles. Fribble lovers are happy. So Zarcons are happy.

Kleene Star Phenomenon

Montagovian First Order Syntax

(every dog (is (a mammal)))

(some dog (is (a mammal)))

(some man (loves (every woman)))

(every (employee-of (a computer-company)) (loves hacking))

(every dog mammal)

(every (loves mary) man)

Montagovian Rules

$$\frac{\texttt{(every } C\ W\texttt{)}}{\texttt{(every (}R\texttt{ (some }C\texttt{)) (}R\texttt{ (some }W\texttt{)))}}$$

$$\frac{\texttt{(every } C\ W\texttt{)}}{\texttt{(every (}R\texttt{ (every }W\texttt{)) (}R\texttt{ (every }C\texttt{)))}}$$

$$\frac{\texttt{(some } C\ W\texttt{)}}{\texttt{(every (}R\texttt{ (every }C\texttt{)) (}R\texttt{ (some }W\texttt{)))}}$$

$$\frac{\texttt{(exists-a (R (some }C\texttt{)))}}{\texttt{(exists-a }C\texttt{)}}$$

$$\frac{\neg\texttt{(exists-a }C\texttt{)}}{\texttt{(every }W\texttt{ (}R\texttt{ (every }C\texttt{)))}}$$

The Kleene Star

If R is a relation expression, then so is R^*.

$$\texttt{(connected-to } n\texttt{)} = \texttt{(neighbor-of}^* \; n\texttt{)}.$$

$$\frac{\texttt{(every (neighbor-of red-node) red-node)}}{\texttt{(every (neighbor-of}^*\texttt{ red-node) red-node)}}$$

Inference Rules for Kleene Star

(every C (R^* C))

(every (R (R^* C)) (R^* C))

$$\frac{\texttt{(every } (R\ C)\ C\texttt{)}}{\texttt{(every } (R^*\ C)\ C\texttt{)}}$$

The Graph Example

Consider a graph with colored nodes such that every arc connects two nodes of the same color. Any two nodes connected by a path of arcs are the same color.

```
(every (a-neighbor-of a-red-node) a-red-node)
```

therefore

$$\texttt{(every (a-neighbor-of}^{*}\texttt{ a-red-node) a-red-node)}$$

Another Maze Example

If a maze containing a rat is placed in a sealed box, then no matter where the rat runs in the maze, it will not get outside of the box.

```
(every (an-adjacent-square a-maze-square)
       a-maze-square)
```

```
(every a-maze-square inside)
```

Process Invariants

- If a frog is placed in a closed box it will remain in the box until the box is opened.

- If Mary is wearing red shoes then she will continue to be wearing red shoes until the shoes are taken off.

- As long as I continue to walk north, I will be north of where I started.

Disjointness Phenomenon

If $f(x) < 6$ and $f(y) > 6$ then $x \neq y$.

If $f(S) < 6$ and $f(W) > 6$ then S and W are disjoint.

Consider a room in which everyone who has a PhD is less than 6 ft tall and everyone who is a Celtics fan is greater than 6 ft tall. If the room contains 3 PhDs and 2 Celtics fans (and nobody else) then there are a total of 5 people.

Consider a tree in which each node is associated with a set. The sets associated with the children of a given node are disjoint subsets of the set associated with the parent. The sets associated with different leaves of the tree are disjoint.

Proof: Consider the least common ancestor k of leaf node n and m. Let n' and m' be the children of k that are ancestors of n and m respectively. Clearly the sets for n' and m' are disjoint supersets of the sets for n and m respectively.

Relational Phenomenon I

```
(define (append l1 l2)
  (if (null?  l1)
      l2
      (cons (car 11) (append (cdr l1) l2))))
```

Append is associative, i.e. (append x (append y z)) equals (append (append x y) z).

```
(define (concat p1 p2)
  (relunion p1
            p2
            (total-rel (domain p1) (domain p2))))
```

A concatenation of disjoint total orders is a total order.

Relational Phenomenon II

A union of disjoint partial orders is a partial order.

A concatenation of disjoint partial orders is a partial order.

Consider a row of marbles. We can “rotate” the marbles by removing the leftmost one and placing at the right end. If we do this as many times as there are marbles then we get the marbles back into their original order.

Constructive Content Phenomenon

A king can reach any square on an empty chess board.

I know how to move a king to any square on an empty chess board.

If y is in $R^*(x)$ then there exists an “R-path” from x to y.

König's Lemma

If a tree has an infinite number of nodes but every node has only a finite number of branches then there exists an infinite branch.

Proof: If there only finitely many branches then some child of the root must be a tree with infinitely many nodes. Pick such a child. This process can be continued indefinitely. QED.

Multiset Example

Let S be a finite multiset of natural numbers. Let R be any relation on natural numbers.

We say that W is an "R-reduction" of S if W can be derived by relacing an element n of S by a finite set of numbers m such that R(m,n).

If W is derived from S by some finite number of such reduction steps then there exists a finitely branching tree such that each node is associated with a number, the numbers for the children of the root node are exactly S, the numbers for the leaf nodes are exactly W, and whenever m is a child of n we have R(m,n).